GeoProof: A user interface for formal proofs in geometry.

Julien Narboux

CNRS, LSIIT, Université de Strasbourg

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Outline

Dynamic Geometry.

There are quite many dynamic geometry software :

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Baghera, Cabri Euclide, Cabri Geometer, CaR, Chypre, Cinderella, Déclic, Défi, Dr. Geo, Euclid, Euklid DynaGeo, Eukleides, Gava, GCLC, GeoExp, GeoFlash, Geogebra, GeoLabo, Geometria, Geometrix, Geometry Explorer, Geometry Tutor, GeoPlanW, GeoSpaceW, GEUP, GeoView, GEX, GRACE, iGeom, KGeo, KIG, Non-Euclid, Sketchpad, Trace en poche, XCas ...

Dynamic Geometry.

But few can deal with proofs :

Baghera, Cabri Euclide, Cabri Geometer, CaR, Chypre, Cinderella, Déclic, Défi, Dr. Geo, Euclid, Euklid DynaGeo, Eukleides, Gava, GCLC, GeoExp, GeoFlash, Geogebra, GeoLabo, Geometria, Geometrix, Geometry Explorer, Geometry Tutor, GeoPlanW, GeoSpaceW, GEUP, GeoView, GEX, GRACE, iGeom, KGeo, KIG, Non-Euclid, Sketchpad, Trace en poche, XCas . . .

1 - Interactive proof systems using a base of lemmas

- Baghera,
- Cabri-Euclide,
- Chypre,
- Défi,
- Geometrix,
- Geometry Tutor

1 - Interactive proof systems using a base of lemmas

2 - Interfaces for an ATP

Cinderella Probalistic method, no proof shown.

GCLC Implementation of the area method, Wu's method and Groebner basis method.

Geometry Explorer Implementation of the full angle method using prolog, and visualization of the proofs in a diagrammatic way.

GEX/Geometer Implementation of the area method, of Wu's method and of deductive database methods, visualization of statements and some visual proofs.

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GeoView Uses GeoPlan and Pcoq to visualize statements .

GeoProof combines these features:

- dynamic geometry
- automatic theorem proving
- interactive theorem proving using a proof assistant (Coq)

What is a proof assistant ?

- The correctness of a proof is decidable by definition.
- A proof assistant is a system to check that a proof is correct.

Motivations

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- We should have both the ability to make arbitrarily complex proofs and use a base of known lemmas.
- The verification of the proofs by the proof assistant provides a very high level of confidence.

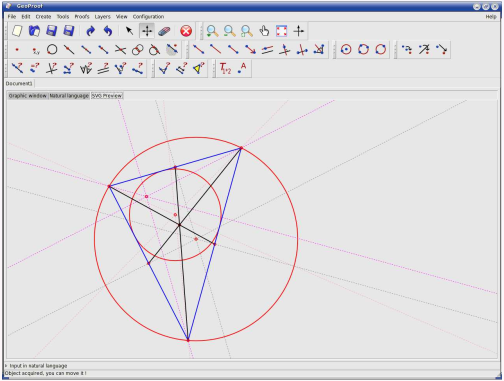
Automated theorem proving in geometry in Coq:

- the area method for euclidean plane geometry
- Gröbner basis method (Loïc Pottier)

A quick overview of GeoProof

A prototype:

- written using `ocaml` and `lablgtk2`,
- distributed under the GPL2 licence,
- multi-platform.



http://home.gna.org/geoproof/

Dynamic geometry features

- points, lines, circles, vectors, segments, intersections, perpendicular lines, perpendicular bisectors,angle bisectors. . .
- central symmetry, translation and axial symmetry
- traces
- text labels with dynamic parts:
  - measures of angles, distances and areas
  - properties tests (collinearity,orthogonality,. . .)
- layers
- Computations use **arbitrary precision**
- Input: XML
- Output: XML, natural language, SVG, PNG, BMP, Eukleides (latex), Coq

Missing features:

- loci and conics
- macros
- animations

Proof related features

1. Automatic proof using an embedded ATP
2. Automatic proof using Coq
3. Interactive proof using Coq

Automatic proof using the embedded ATP

We need to perform a translation from a theory based on circles, lines and points to a theory based only on points.

l passing through A and B $\mathcal{P}_1(l) = A$ $\mathcal{P}_2(l) = B$

l parallel line to m passing through A $\mathcal{P}_1(l) = A$ $\mathcal{P}_2(l) = P2_l$

l perpendicular to m passing through A $\mathcal{P}_1(l) = A$ $\mathcal{P}_2(l) = P2_l$

l perpendicular bisector of A and B $\mathcal{P}_1(l) = P1_l$ $\mathcal{P}_2(l) = P2_l$

c circle of center O passing through A $\mathcal{O}(c) = O$ $\mathcal{P}(c) = A$

c circle passing through A,B and C $\mathcal{O}(c) = O_c$ $\mathcal{P}(c) = A$

c circle whose diameter is A B $\mathcal{O}(c) = O_c$ $\mathcal{P}(c) = A$

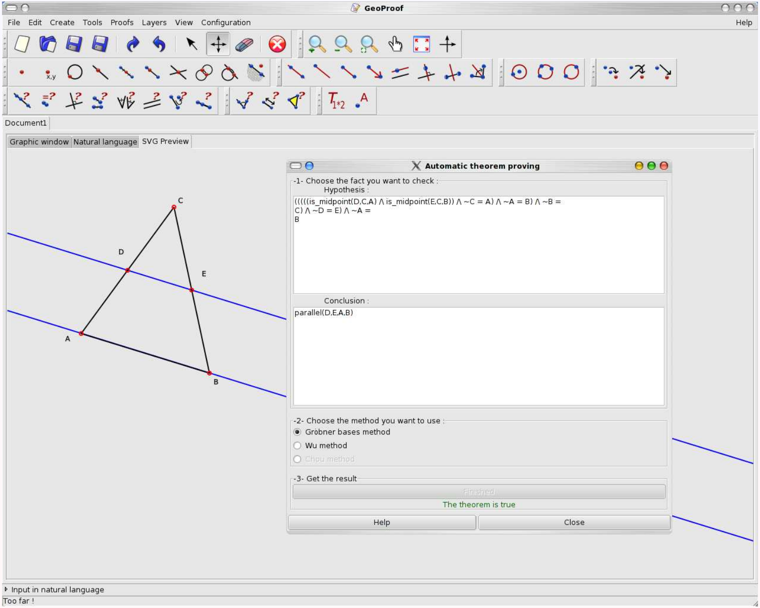
Point P on line l $collinear(P, \mathcal{P}_1(l), \mathcal{P}_2(l))$

l intersection of l_1 and l_2

$$collinear(I, \mathcal{P}_1(l_1), \mathcal{P}_2(l_1)) \wedge$$
$$collinear(I, \mathcal{P}_1(l_2), \mathcal{P}_2(l_2)) \wedge$$
$$\neg parallel(\mathcal{P}_1(l_1), \mathcal{P}_2(l_1), \mathcal{P}_1(l_2), \mathcal{P}_2(l_2))$$

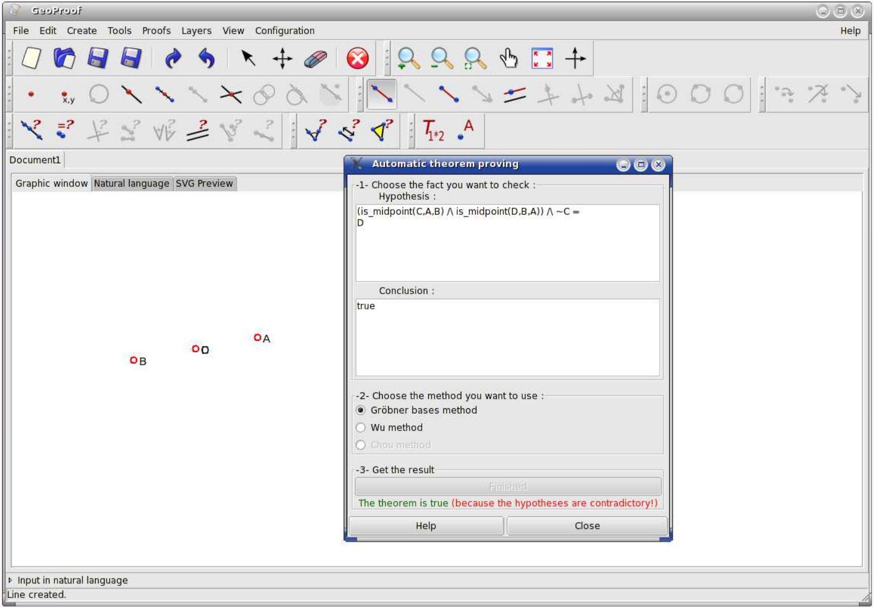
l perpendicular bisector of AB

$$\mathcal{P}_1(l)A = \mathcal{P}_1(l)B \wedge \mathcal{P}_2(l)A = \mathcal{P}_2(l)B \wedge$$
$$\mathcal{P}_1(l) \neq \mathcal{P}_2(l) \wedge A \neq B$$


GeoProof
File Edit Create Tools Proofs Layers View Configuration
Help
Document1
Graphic window
Natural language
SVG Preview
Automatic theorem proving
-1- Choose the fact you want to check :
Hypothesis :
(((((is_midpoint(D,C,A) ∧ is_midpoint(E,C,B)) ∧ ~C = A) ∧ ~A = B) ∧ ~B = C) ∧ ~D = E) ∧ ~A = B
Conclusion :
parallel(D,E,A,B)
-2- Choose the method you want to use :
Gröbner bases method
Wu method
Chou method
-3- Get the result
Finished
The theorem is true
Help
Close
C
D
E
A
B
Input in natural language
Too far !

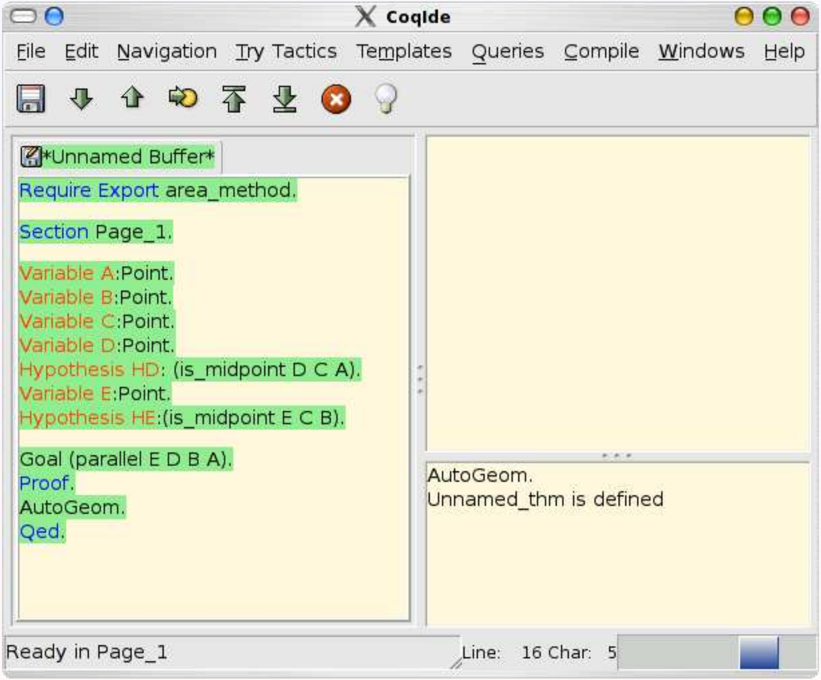
Dealing with non-degeneracy conditions

- Non degeneracy conditions play a crucial role in formal geometry.
- GeoProof allows to build a formula not a model of this formula.
- The user can define impossible figures.

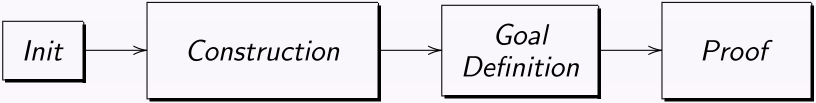

GeoProof
File Edit Create Tools Proofs Layers View Configuration Help
Document1
Graphic window Natural language SVG Preview
A
D
B
Automatic theorem proving
-1- Choose the fact you want to check :
Hypothesis :
(is_midpoint(C,A,B) /\ is_midpoint(D,B,A)) /\ ~C = D
Conclusion :
true
-2- Choose the method you want to use :
Gröbner bases method
Wu method
Chou method
-3- Get the result
Finished
The theorem is true (because the hypotheses are contradictory!)
Help
Close
Input in natural language
Line created.

Automatic proof using Coq

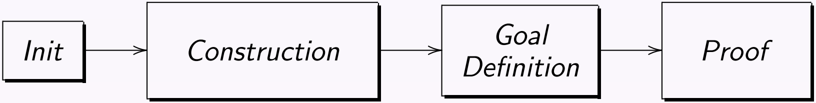
- Based on our formalization of the area method in Coq.
- Constructive theorems in euclidean plane geometry.


CoqIde
File Edit Navigation Try Tactics Templates Queries Compile Windows Help
Unnamed Buffer
Require Export area_method.
Section Page_1.
Variable A:Point.
Variable B:Point.
Variable C:Point.
Variable D:Point.
Hypothesis HD: (is_midpoint D C A).
Variable E:Point.
Hypothesis HE:(is_midpoint E C B).
Goal (parallel E D B A).
Proof.
AutoGeom.
Qed.
AutoGeom.
Unnamed_thm is defined
Ready in Page_1
Line: 16 Char: 5

Interactive proof using Coq

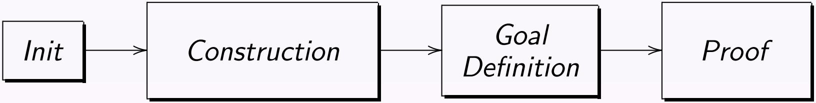


Interactive proof using Coq



- GeoProof loads the library (axioms and theorems) and updates the interface.

Interactive proof using Coq



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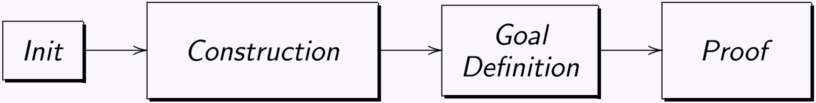
- It translates each construction as an hypothesis in Coq syntax.

Interactive proof using Coq



- GeoProof loads the library (axioms and theorems) and updates the interface.
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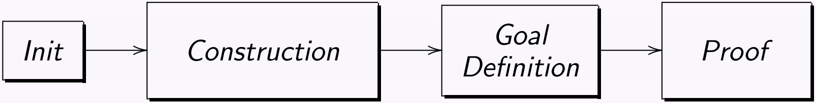
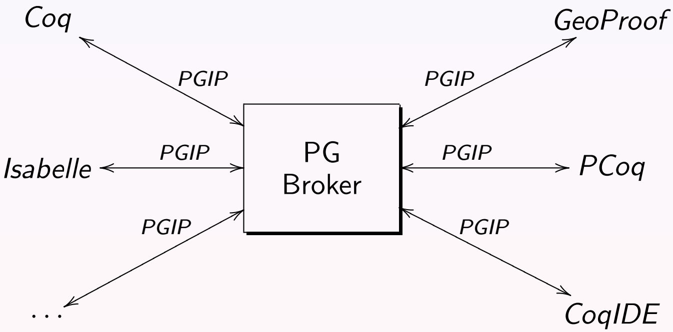
- It translates the conjecture into Coq syntax.

Interactive proof using Coq



- GeoProof loads the library (axioms and theorems) and updates the interface.
- It translates each construction as an hypothesis in Coq syntax.
- It translates the conjecture into Coq syntax.
- It translates each construction into the application of a tactic to prove the existence of the newly introduced object.

A possible framework for synchronization between GUIs



Another approach

Integrating the proof checker, the proving GUI, and the dynamic geometry software in a single window.

Future work

- Diagrammatic proofs
  - in geometry and
  - in abstract term rewriting.
- Tighter integration between the gui and proof assistant.
- Two ways communication between the proof assistant and the DGS.

My wishes

- A language/API to export/import statements.
- Statements should be relative to an axiom system.
- Statements should not impose a geometric construction.
- Non degeneracy conditions should not be overlooked.

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